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Antonio de' Mazzinghi: An Algebraist of the 14th Century*

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The three major encyclopedias of medieval mathematics (Pal.573, Ott.lat.3307, L.IV.21) acclaim Antonio de' Mazzinghi as the best algebraist of the 14th and 15th centuries. This paper presents Antonio's biography and an analysis of his algebraic work. © 1988 Academic Press, Inc.

Die drei grösseren Enzyklopädien der Mathematik des Mittelalters (Pal.573, Ott.lat.3307, L.IV.21) vertreten übereinstimmend die Ansicht, dass Antonio de' Mazzinghi der beste Algebraiker des vierzehnten und fünfzehnten Jahrhunderts war. Diese Arbeit enthält eine Biographie von ihm und eine sorgfältige Analyse seiner algebraischen Werke. © 1988 Academic Press, Inc.

Le tre maggiori enciclopedie della matematica medioevale (Pal.573, Ott.lat.3307, L.IV.21) sono concordi nell'affermare che Antonio de' Mazzinghi fu il miglior algebrista del 14° e 15° secolo. Questo lavoro presenta la sua biografia ed un'analisi accurata della sua opera algebrica. © 1988 Academic Press, Inc.

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1. INTRODUCTION

In Italy during the 14th and 15th centuries, algebra was cultivated in the abacus schools (*Scuole d'abaco*), whose primary task was to teach practical arithmetic. Many abacus teachers (*Maestri d'abaco*) wrote texts, called *Trattati d'abaco*, which usually contained a chapter devoted to algebra.

Three manuscripts of the mid-15th century are among the most important documents for studying the development of algebra in Italy in the above-mentioned centuries. These texts may be considered as very detailed encyclopedias of the mathematical knowledge taught in Italian abacus schools. They record the names of the best known abacus teachers, give short biographic notes on the most famous of them, and present long excerpts from their works.

The manuscripts in question are Palatino 573 (ca. 1460), the Florence National Library; Ottoboniano latino 3307 (ca. 1465), the Vatican Apostolic Library; and L.IV.21 (1463), the Siena Municipal Library [Arrighi 1965, 1967, 1968].

These manuscripts acclaim Antonio de' Mazzinghi as the best algebraist of the 14th and 15th centuries and quote large parts of his works.

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In a recent paper L. Toti Rigatelli and I examined the history of algebra from Leonardo of Pisa to Luca Pacioli and established a continuity of algebraic development between these two authors [Franci & Toti Rigatelli 1985]. In particular, we proved that many results that historians of mathematics had attributed to Luca Pacioli were already known two centuries earlier. Among the authors who greatly improved algebra in the 14th century was Antonio de' Mazzinghi, and thus we briefly examined his algebraic work.

In the present paper, a careful analysis of Antonio's surviving works and a comparison with those of the other authors of the 14th and 15th centuries allow us to confirm the judgment of the ancient Florentine masters.

2. THE LIFE AND WORKS

The little information we have about Antonio de' Mazzinghi's life is contained in the above-mentioned encyclopedias, written more than 70 years after his death. Moreover, some Florentine archival sources record the names of many members of his family.

The Mazzinghi family came originally from Peretola, a small town near Florence. One of its members, Bene di Spinello, a wealthy merchant, was first listed on the rolls of the silk merchants' guild in 1351 and was prior of the city in 1381 [Boncompagni 1854, 134–137].

Antonio was born around 1353 and it seems likely that he was the son of Bene di Spinello. According to Benedetto, Antonio's father took particular care of his education because he wanted his son to acquire "virtues which could not be taken away by any accident" (*virtù le quali per alcuno accidente gli fusseno tolte*). Having rapidly learned to read and write Italian and Latin, Antonio began studying mathematics under the guidance of Paolo dell'Abaco, who was at the time the most learned Florentine abacist. The same Paolo was probably a friend of Antonio's father; in fact Bene di Spinello was one of the executors of Paolo's will [Van Egmond 1977]. Paolo died in 1367, so Antonio could benefit from his teaching for a short time only.

Nevertheless, as already noted, all three encyclopedias claim that Antonio was the best Florentine mathematician of all times. To support their claim, they tell a very enlightening story from Antonio's life. In his will Paolo directed that all his astrological books and instruments be placed in a case until a committee of four masters chose the most learned mathematician in Florence. After a long discussion (which ran five years) among the judges, the books and instruments were given to the young Antonio, who was then teaching in the *Bottega d'abaco di Sancta Trinita*, founded by Paolo himself [Arrighi 1965]. Antonio died very young in 1383.

Benedetto says Antonio was very learned in arithmetic, geometry, astrology, building, perspective, and, above all, algebra. Although the three encyclopedias agree that Antonio was most learned in algebra, they still remember him as the first person to calculate tables of interest (*le tavole del merito*).

Benedetto recalls also that Antonio wrote many treatises on arithmetic and geometry, the best of which was his *Fioretti,* while Domenico d'Aghostino's student repeatedly mentions Antonio's treatise on algebra. The latter affirms also that copies of Antonio's treatises were, in this time, in the possession of the Florentine abacist Lorenzo di Biagio. Unfortunately, none of the numerous known manuscripts of the 14th and 15th centuries containing treatises on arithmetic, geometry, or algebra is attributed or attributable to Antonio.

All that at present survives of his works is what is quoted in the above-mentioned encyclopedias as well as in the Magliabechiano Cl.XI,120 of the Florence National Library. In particular, folios 451r–474v of L.IV.21 contain 45 problems from the treatise *Fioretti,* folios 398r–402v of Pal.573 quote the first part of the treatise of algebra, folios 478v–409r of Pal.573 contain 22 problems, folios 335r–343r of Ott.lat 3307 contain 15 problems, and finally folios 7v–10v of Magl. Cl.XI,120 are devoted to the "rules of the new algebra found by Maestro Antonio of Florence presented in 9 chapters" (*Regole dell'arzibra nuova truovate per Maestro Antonio da Firenze in 9 capitoli*), followed by 6 problems.

Fortunately, we are reasonably sure of the trustworthiness of the sources through which Antonio's mathematical works have reached us. The authors who have handed parts of his works down to us seem to have been faithful transcribers. First it should be noted that the authors of L.IV.21 and Pal.573 claim that they have copied directly from Antonio's treatises. Furthermore, a comparison of the problems that are quoted in all the above-cited manuscripts shows them to be substantially alike and thus confirms their faithfulness to the source. Finally it can be noted that both Pal.573 and L.IV.21 quote many problems from Chapter 15 of *Liber abaci.* A comparison with the original, which we also know from other sources, again confirms the essential reliability of the transcriptions in the manuscripts in question.

3. THE *TRATTATO D'ALGEBRA*

Taken together, the above-mentioned excerpts give enough material for us to reconstruct, with a good approximation, the subject and the structure of Antonio's treatise of algebra. The treatise certainly contained at least three parts devoted to: operations with monomials and polynomials, rules for solving nine types of equations, and problems solved by algebra.

Prior to an explanation of the content of the treatise perhaps it is appropriate to recall that in medieval and Renaissance algebra the unknown and its powers were represented by words; similarly equations and rules for solving them were written out in words.

I. The first part of the treatise, as reported in Pal.573, begins with the following definitions:

> In this part *cosa* (thing) is an unknown quantity, *censo* is the square of the mentioned *cosa*, *chubo* is the product of *cosa* by *censo*, *censo di censo* is the square of *censo* or the product of *cosa* by *chubo*. And remark the terms of algebra are all in the continuous proportion as: *cosa*, *censo*, *chubo*, *censo di censo*, *chubo relato*, *chubo di chubo* and so on. [Pal.573, c.339r]

Next, some rules for multiplication among terms of algebra are presented. In modern symbolism they are: $xx = x^2$, $xx^2 = x^3$, $xx^3 = x^4$, $xx^4 = x^5$, $xx^5 = x^6$, $x^2x^2 =$

x^4, $x^2x^3 = x^5$, and $x^2x^4 = x^6$. Some rules for multiplying numbers and powers of the unknown follow, with particular care being devoted to the case in which numerical coefficients are radicals.

The treatise next shows how to divide the unknown and its powers up to the sixth, one by another, in all the possible ways. This subject is presented in a systematic and detailed fashion: in each case the general rule and a numerical example are given. In this context Antonio introduces the fractions $1/x$, $1/x^2$, . . . , $1/x^6$ which are represented as: 1/*cosa-esimi,* 1/*censo-esimi*, . . . , 1/*chubo di chubo-esimi*.

Only two simple examples are devoted to the multiplication of polynomials. More interesting is the following part regarding division of polynomials.

> And when you must divide one or more terms of algebra, make the fraction putting above what you must divide, and below what is the divisor; as when you wish to divide 3 *censi* by 2 *cose* and 3 *chubi*, we say that it makes this fraction 3 *censi*/(2 *cose* and 1 *chubo-esimi*). . . . And if you wish to divide two or more terms of algebra by only one term, then a fraction can arise, and also it can become a number or a term of algebra. As when you wish to divide 3 *cose* and 8 *censi* and 18 *chubi* by 3 *censi*. As divided before, from 3 *cose* by 3 *censi* comes this fraction, i.e., 1/*cosa-esimi*, and from 8 *censi* divided by 3 *censi* comes the number 2+2/3, and from 18 *chubi* divided by 3 *censi* comes 6 *cose*. And thus you have for this result 6 *cose* and 2+2/3 as a number and this fraction 1/*cosa-esimi*. [Pal.573, c.402v]

II. Folios 7v and 8r in Magl.Cl.XI,120 allow us to imagine the content of the second part of Antonio's treatise. Here nine types of equations and related rules of resolution are presented. They are divided into two groups: simple (the first six) and compound (the last three).

In medieval and Renaissance algebra, equations were formed equalizing two polynomials; when there was a monomial on each side, the equation was said to be simple, otherwise it was called compound. Furthermore, equations and the rules to solve them were written in words. Thus in the following list of the nine equations we have translated the original rhetorical expressions into their modern equivalents, adding symbols for coefficients, which in medieval rules were only implied.

1. $ax = c$, 2. $ax^2 = c$, 3. $ax^3 = c$,
4. $ax^4 = c$, 5. $ax^5 = c$, 6. $ax^6 = c$,
7. $ax^2 = bx + c$, 8. $ax^2 + bx = c$, 9. $ax^2 + c = bx$.

The rules given to solve the above-listed equations are the exact modern ones, taking into account that in those times only real and positive solutions were calculated. However, to give an idea of the way they were presented, we give the translation of rules 3 and 9.

> 3. When the *chubi* are equal to the number you must divide the number by [the coefficient of] *chubi* and the cubic root of what results is equal to the *cose*.
> 9. When the *censi* and the number are equal to the *cose*, you must divide by [the coefficient of] *censi*, and then halve the [coefficient of the] *cose* and multiply it by itself and take away the number, and the half of the *cose* minus or plus the root of what remains is equal to the *cosa*. [Magl.Cl.XI,120, cc.7v–8r]

TABLE I
CORRESPONDENCE AMONG ANTONIO'S PROBLEMS IN Pal.573, Ott.lat.3307, Magl.Cl.XI, 120, AND L.IV.21 [1]

Pal. 573	Ott.lat.3307	Magl.Cl.XI, 120
1 (~9)	1 (7)	1 (12)
2 (~10)	2 (8)	2 (13)
3 (11)	3 (9)	3
4	4 (~10)	4 (6)
5 (12)	5 (11)	5 (9)
6 (13)	6 (12)	6 (15)
7 (14)	7 (13)	
8 (15)	8 (14)	
9 (16)	9 (15)	
10 (18)	10 (16)	
11 (7)	11 (18)	
12 (~20)	12 (~20)	
13 (21)	13 (21)	
14	14 (~22)	
15 (8)	15 (~4)	
16		
17		
18		

Rules 1, 2, 7, 8, and 9 are those presented in the treatises of algebra by al-Khwarizmi and Leonardo of Pisa, while rules 3, 4, and 5 are not explicitly stated by these authors, who give instead the rule for $ax^2 = bx$. In the treatises of the above-mentioned authors rules 7, 8, and 9 are justified by geometrical proofs. In Magl.Cl.XI, 120 these proofs are lacking; however, it is impossible to say if this gap also occurs in Antonio's text or if it is attributable to the manuscript author.

III. To reconstruct the last part of the treatise, which contains the problems, we can draw from all the above-cited manuscripts. Unfortunately there is a remarkable duplication of problems from one manuscript to another, so we have only 55 different problems (for the correspondence of the problems in the various manuscripts, see Table I).

All the problems that have been handed down to us are very difficult, and thus we cannot exclude the hypothesis that in the original text there were more elementary problems. It is possible that the manuscript's authors did not quote the elementary problems because they belonged to the common tradition and so did not show Antonio's personal contribution.

The majority of the problems attributed to Antonio are of a theoretical type; they involve finding two or three numbers satisfying some conditions or dividing a given number into two or three parts—in a few cases, four or five parts. Only 11 problems are of a commercial type. Precisely, there are 3 on the barter of goods, 3 on the exchange of money, 3 on the calculation of interest, and one on partnership and alligation (i.e., the way to calculate the amount of gold or silver in a coin).

4. ANALYSIS OF SOME PROBLEMS

In this section we examine in detail some problems evidencing Antonio's algebraical skill.

All the problems analyzed here are taken from those presented by Benedetto of Florence in L.IV.21. This collection in fact was printed some years ago, so it is more easily available [Antonio 1967]. Also the numbering of the cited examples refers to the aforementioned collection.

In the resolution of the first three problems presented by Benedetto, Antonio shows his algebraic ability to reduce one problem to another.

1. Make 3 continuous proportional parts of 19 such that the first multiplied by the other 2 and the second by the other 2 and the third by the other 2, and added together their sum makes 228. [Antonio 1967, 15]
2. Find 3 numbers in a continuous proportionality, i.e., 3 quantities in a continuous proportionality, such that the first multiplied by the other 2 and the second by the other 2 and the third by the other 2, their sum makes 888, and, each part multiplied by itself, those squares added together make 481. [Antonio 1967, 18]
3. Make 3 continuous proportional parts of 9 + 1/2, such that their squares are 33 + 1/4, i.e., the square of the first added with the square of the second with the square of the third, makes 33 + 1/4. [Antonio 1967, 19]

In modern algebraic symbolism, the solutions of these problems are equivalent respectively to those of the following systems:

$$(1)\ \begin{cases} x + y + z = 19 \\ x : y = y : z \\ x(y + z) + y(x + z) + z(x + y) = 228 \end{cases}$$

$$(2)\ \begin{cases} x : y = y : z \\ x(y + z) + y(x + z) + z(x + y) = 888 \\ x^2 + y^2 + z^2 = 481 \end{cases}$$

$$(3)\ \begin{cases} x + y + z = 9 + 1/2 \\ x : y = y : z \\ x^2 + y^2 + z^2 = 33 + 1/4. \end{cases}$$

In solving problem 1, Antonio remarks that since $y^2 = xz$, one has $x(y + z) + y(x + z) + z(x + y) = 2y(x + y + z)$. So by the third condition it follows that $2y \times 19 = 228$. Once the second part is found, he easily calculates the others.

Before solving problem 2, Antonio proves in a geometrical way the identity: $(x + y + z)^2 = x^2 + y^2 + z^2 + x(y + z) + y(x + z) + z(x + y)$. Using this identity, he calculates $(x + y + z)^2 = 888 + 481$, from which he obtains $x + y + z = 37$. Then he remarks that a problem similar to the previous one must now be solved.

In the solution of problem 3, Antonio recalls the identity proved in problem 2 and calculates $x(y + z) + y(x + z) + z(x + y) = (9 + 1/2)^2 - 33 + 1/4 = 57$. Thus he again turns the solution of his problem into that of a problem similar to problem 1.

One of the most important moments in solving a problem by algebra is surely that of choosing the unknown quantity. There follow some problems where the choice made by Antonio seems to be particularly opportune.

34. Divide 10 into 2 parts such that the one divided by the other and the other by the one and each quotient multiplied by itself and added together, what they make divided by the sum of the above results comes to 3 + 53/68. [Antonio 1967, 70]

The problem asks for two numbers u and v such that $u + v = 10$ and $((u/v)^2 + (v/u)^2)/(u/v + v/u) = 3 + 53/68$. Antonio puts $u/v + v/u$ equal to one *cosa*. Then recalling that $(u/v)(v/u) = 1$, he calculates u/v and v/u as the solutions of the equation $t^2 - xt + 1 = 0$. The value of the *cosa* is found by using the second condition.

The same choice of the unknown is made in problems 35, 36, and 37, which are equivalent to the following systems:

$$(35) \begin{cases} u + v = 10 \\ \left(\frac{u}{v} + \frac{v}{u}\right) - \left(\frac{u}{v} + \frac{v}{u}\right) = 15 + 15/16 \end{cases} \qquad (36) \begin{cases} u + v = 12 \\ \left(\frac{u}{v}\right)^2 + \left(\frac{v}{u}\right)^2 + \left(\frac{u}{v} - \frac{v}{u}\right) = 6 + 1/2 \end{cases}$$

$$(37) \begin{cases} u + v = 12 \\ \left[\left(\frac{u}{v}\right)^2 + \left(\frac{v}{u}\right)^2\right]\left(\frac{u}{v} - \frac{v}{u}\right) = 9 + 9/16. \end{cases}$$

38. Divide 12 into two parts such that the one divided by the other and the other by the one and added together the result is equal to 5/64 of the multiplication of the first part in the second. We ask, what are the parts? Put that the multiplication of the one part in the other is one *cosa*, then to find the parts we must divide 12 into 2 parts such that the one multiplied by the other makes one *cosa*, ... [Antonio 1967, 76]

It is thus proposed to find two numbers u and v such that $u + v = 12$ and $u/v + v/u = 5/64\ (uv)$. Antonio puts uv equal to one *cosa* and then he calculates u and v as the solutions of the equation $t^2 - 12t + x = 0$. The other condition allows him to find the value of x.

A similar choice of the unknown is made in problems 39 and 40, which are equivalent to the systems:

$$(39) \begin{cases} u + v = 15 \\ uv = 10 + 16 \Big/ \left(\frac{u}{v} + \frac{v}{u}\right) \end{cases} \qquad (40) \begin{cases} u + v = 14 \\ \sqrt{u^2 + v^2} = (u - v)^2 - 6. \end{cases}$$

In solving many problems Antonio even uses two unknowns, one called *cosa* and the other *quantità*. As far as I know, Antonio is the first algebraist to use two unknowns.

10. Find two numbers such that the sum of their squares is 100, and multiplying one by the other makes the square of the difference of the two numbers minus 5. [Antonio 1967, 30]

It is a matter of finding two numbers u and v such that $u^2 + v^2 = 100$ and $uv = (u - v)^2 - 5$. Antonio begins the resolution:

> Let the first number be one *cosa* plus the root of some *quantità*, and the second be one *cosa* minus the root of the same *quantità*, and multiply each number by itself and add the squares, it makes 2 *censi* and some unknown *quantità*. And those squares must make 100, where that unknown *quantità* is the difference that is from 100 to 2 *censi*, that is, 100 minus 2 *censi*. Thus the first multiplication, i.e., the *quantità* is 50 minus 1 *censo*. The first number is one *cosa* plus the root of 50 minus 1 *censo* and the second is one *cosa* minus the root of 50 minus 1 *censo*. [Antonio 1967, 30]

That is, Antonio puts $u = x + \sqrt{q}$ and $v = x - \sqrt{q}$, then $100 = u^2 + v^2 = 2x^2 + 2q$ and $q = 50 - x^2$. Thus he has $u = x + \sqrt{50 - x^2}$ and $v = x - \sqrt{50 - x^2}$. Next he calculates $u - v = 2\sqrt{50 - x^2}$, $uv = 2x^2 - 50$, and finally obtains the value of x by the second condition.

Two unknowns are used also in problems 9, 18, 19, 20, 21, and 22. All these cases involve finding two numbers whose product or sum of squares is known. When $uv = N$ is known, Antonio sets $u = q + x$ and $v = q - x$ and calculates $N = uv = q^2 - x^2$. Thus he gets $q = \sqrt{x^2 + N}$ and $u = \sqrt{x^2 + N} + x$, $v = \sqrt{x^2 + N} - x$. In some cases $N = (u - v)^2$, so $N = 4x^2$ and $q = \sqrt{5x^2}$.

Very often the problems Antonio proposes are of such a kind that, in spite of a good choice of the unknown, the conditions lead to involved expressions, so that to reach one of the nine canonical forms it is necessary to make a lot of calculations. Among these calculations are sums and products of polynomials, sums and quotients of algebraic fractions, and rationalization of radicals containing the unknown. These operations are obviously made in the rhetorical way, but with such clarity and algebraic taste that it is sufficient to translate the words into modern symbolism to ascertain that the calculations are made just as we would make them today, as can be seen in the resolution of the following problem.

> 27. Make two parts of 10 such that the one divided by the other and the other by the one and 16 divided by each of these results and the roots taken and added together makes 10. [Antonio 1967, 59]

It is proposed to find two numbers u and v such that $u + v = 10$ and $\sqrt{16/(u/v)} + \sqrt{16/(v/u)} = 10$. Antonio puts $u = 5 - x$ and $v = 5 + x$ (*farai positione che l'una parte sia 5 meno una cosa e lla sechonda sia 5 più una cosa*). Then he calculates u/v and v/u obtaining $((5 - x)/(5 + x))$ ((*5 meno 1 cosa*)/(*5 più 1 cosa*)) and $((5 + x)/(5 - x))$ ((*5 più 1 cosa*)/(*5 meno 1 cosa*)). Next he divides 16 by these fractions obtaining $((80 + 16x)/(5 - x))$ ((*80 più 16 cose*)/(*5 meno 1 cosa*)) and $((80 - 16x)/(5 + x))$ ((*80 meno 16 cose*)/(*5 più 1 cosa*)). The calculations go on as follows:

> And of these results we have to take the roots and to add them together and they must make 10. Therefore taking the root we have the root of (80 p.16 co.)/(5 m. 1 co.) and the root of (80 m. 16 co.)/(5 p. 1 co.) and these make 10. And we know that multiplying each of them by itself and twice one by the other will make as to multiply 10 by itself. . . . And you must add together these two multiplications, i.e., you must add together (80 m. 16 co.)/(5 p. 1 co.) with (80 p. 16 co.)/(5 m. 1 co.), . . . it comes (800 p. 32 ce.)/(25 m. 1 ce.). And subtract this result from 100, it remains 100 minus this, i.e., (800 p. 32 ce.)/(25 m. 1 ce.). But still it can be given in one quantity, and this is that you change 100 to this fraction, . . . , it remains (1700 meno 132 ce.)/(25 meno 1 ce.). And this is equal to twice the multiplication of one part by the other. And multiplying the root of (80 m. 16 co.)/(5 p. 1 co.) by the root of (80 p. 16 co.)/(5 m. 1 co.) makes

> the root of (6400 m. 256 ce.)/(25 m. 1 ce.). And this quantity is equal to the half of (1700 meno 132 ce.)/(25 meno 1 ce.), i.e., to (850 meno 65 ce.)/(25 meno 1 ce.). Hence to take off the root you shall multiply each quantity by itself. . . . [Antonio 1967, 60]

At the end of his calculations Antonio obtains the equation $4100x^4 + 562{,}500 = 99{,}400x^2$, which does not coincide with any of the nine types listed above. However, he calculates x^2 following rule 9. Also problems 35 and 37 lead to solutions of equations of this kind.

5. CONCLUSIONS

The analysis of the previous sections, even though far from complete, shows Antonio de'Mazzinghi as a skilled master of algebraic devices. His ability is still more enhanced if we compare his work with that of the other algebraists of the 14th and 15th centuries.

Study of the treatises on algebra of the aforementioned centuries [2] shows that algebra underwent great development in Italy in the 14th century [Franci & Toti Rigatelli 1988]. This development was first characterized by a process of arithmetization, i.e., of application of techniques of the arithmetical calculus to algebraic quantities.

The chapter on algebra of Leonardo Fibonacci's *Liber abaci* (1202) and of most of the treatises of the same period lacked a section devoted to algebraic calculations. These calculi were taught in the course of the solution of the problems. Leonardo often avoided algebraic calculations and substituted for them geometric solutions, which were obtained by setting the quantity in question equal either to a segment or to an area. A typical algebraic calculation that Leonardo systematically approached with the methods of geometry was the sum of algebraic fractions.

The section of Antonio's treatise devoted to algebraic calculations was the most complete among those written in the period under consideration. Thus Antonio was doubtless one of the principal authors of the above-mentioned process of arithmetization.

Finally comparative study of the best collections of problems solved by algebra written in the 14th and 15th centuries again shows the algebraic superiority of Antonio [3], not only because the problems Antonio proposed and solved were more difficult, but because he showed a greater algebraic intuition. As we have seen, this intuition manifested itself above all in the opportune choice of the unknown, in turning one problem into another already solved, and in singling out classes of similar problems.

NOTES

1. The numbers in brackets refer to L.IV.21 numeration. If a number is preceded by ~, it means that the problem in L.IV.21 is of the same type but it has different numerical coefficients. We point out that problems 2 and 12 in Pal. 573 have different numerical coefficients respectively from problems 4 and 12 in Ott.lat. 3307.

2. Laura Toti Rigatelli and I have studied 75 of these treatises, 15 of which are of the 14th century. The latter are listed in [Franci & Toti Rigatelli 1988]. A list of most of the others is available in [Franci & Toti Rigatelli 1985].

3. Some of these collections have been printed [Biagio (14th c.) 1983; Giovanni di Bartolo (15th c.) 1982; Luca di Matteo (15th c.) 1986].

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